THE PREDICTION AND MEASUREMENT OF SOUND RADIATED BY STRUCTURES

Richard H. Lyon and J. Daniel Brito
Massachusetts Institute of Technology

INTRODUCTION

This paper is a review of certain theoretical ideas about the radiation of sound and shows how these ideas have been implemented in strategies for explaining or measuring the sound produced by practical structures. We shall be especially interested in those aspects of the subject that relate to the determination of the relative amounts of sound generated by various parts of a machine or structure, which can be very useful information for noise reduction efforts. We will also point out areas in which significant uncertainties or questions remain in the theoretical and experimental aspects of the subject.

Intensity and Energy Density

Since the acoustical equations are first-order perturbations of the underlying fluid, dynamical, and state equations, it is not obvious that acoustical intensity, which is a second-order quantity, can be determined from the first-order quantities only. It is shown in advanced texts, however, that in a nonmoving ideal fluid, an energy conservation statement can be written in the form

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot \vec{I} = 0, \tag{1}$$

where $\varepsilon = \frac{1}{2}\rho_0 u^2 + \frac{1}{2}p^2/\rho_0 c^2$ is the acoustical energy density, $\vec{I} = p\vec{u}$ is the intensity, p and $\vec{u}$ are the first-order "acoustic" pressure and particle velocity respectively. This formulation, while internally consistent, does leave certain second-order terms out of the intensity and energy density which correspond to the transport of internal energy of the fluid by streaming flows. These terms are usually of little practical importance.

From eq. (1), it is clear that the addition to $\vec{I}$ of any solenoidal vector field will leave the conservation relation unchanged, but can greatly alter the intensity vector at any position (and time). Such solenoidal intensity fields do exist in reverberant fields (even when time averaged) and represent one way that reverberant sound can contaminate a measurement of sound intensity.

Simple Radiators

A simple radiator is a rigid plane surface, vibrating with a velocity u in a direction perpendicular to its own surface, set in an infinite rigid plane. When all dimensions of this vibrator are large compared to a wavelength, then the magnitude of the intensity is $I = u^2 \rho_o c$. If the vibrator is a circular piston of radius a, then the total time-averaged radiated-sound power is

$$\Pi_{rad} = \langle u^2 \rangle \rho_o c S \sigma_{rad} \tag{2}$$

where S is the area of the piston and σ_{rad} is the radiation efficiency, shown graphed in fig. 1.

The interesting feature of fig. 1 is that, as expected, σ_{rad} approaches unity at frequencies such that the wavelength is small compared to piston diameter, but also, this limit is essentially reached when the piston diameter is only about one-third of the wavelength of the sound wave. This geometric effect is very important in sound radiation by machines since many machines have sizes comparable to the wavelength of sound at frequencies of interest.

The radiation of sound by waves on a plane can be pictured as shown in fig. 2. Above the critical frequency, the flexural waves become supersonic (acoustically fast), and there is highly directed sound radiation. Below this frequency, the flexural wave is subsonic (acoustically slow), and there is no sound radiation from an infinite plate. The critical frequency is determined by elastic properties of the plate -- a simple formula for steel, aluminum, or glass is:

$$f_c = 500/h \text{ (in)}$$

where the thickness h is expressed in inches. This effect of bending wave-speed on radiation efficiency is very important for large flat structures, but less so for highly curved, segmented, or stiffened structures. Plate damping generally has a large effect on the amplitude of vibration (determines $\langle u^2 \rangle$), but has little practical effect on the radiation efficiency.

Above the critical frequency, the theoretical sound intensity is uniform over the surface. This is also the case for large finite supported plates, as shown in fig. 3. With a single mode of vibration, there are nodal lines in the intensity that correspond to zero velocity node lines on the plate. When the vibration is multimodal, the intensity pattern becomes more uniform.

Below the critical frequency, any interruption or discontinuity in the properties of an infinite plate (such as the line of support shown in fig. 4) will result in radiation of sound at frequencies less than the critical frequency. The normal component of intensity for such a support due to flexural waves reflecting from it at normal incidence is shown in fig. 4. Note the alternating regions of positive and negative intensity, with the region next to the support line being positive. The net radiation due to the line support is, of course, positive, but various regions of the plate are emitting and absorbing sound over an extended area.

When the plate is finite, it has been shown that below the critical frequency, the radiation efficiency is proportional to the perimeter of the plate. This has led to the concept of "edge radiation", with the strong implication that the sound is radiated from the edges of the plate. The average radiation efficiency for a supported plate is shown in fig. 5. A direct calculation of the sound intensity for a supported rectangular plate shows that the region very close to the edge is nearly always radiating, particularly for the edge modes that have trace wavelengths greater than the wavelength of sound. In fig. 6, we show a scan of intensity across a section of a simply supported plate for a single mode of vibration mode (17, 1), which clearly shows this edge radiation effect. In fig. 7, the case of multimodal radiation is shown. Thus, although the total radiated sound power from the plate is proportional to its edge length, the *pattern* of intensity is more like that of a set of surface radiators and absorbers with a line of radiation along the edge.

In addition to radiator size, plate thickness, and framing or support structure effects, curvature effects can also play an important role in radiation efficiency. Curved surfaces are stiffer than flat structures and may vibrate less, but their radiation efficiencies are generally higher. In fig. 5, we show the effect of curvature by comparing the radiation efficiency of a flat supported plate with that of a cylinder formed by rolling the plate.

EXPERIMENTAL METHODS

A number of experimental techniques are available for determining the amount of sound power radiated by a structure (or machine). Some methods simply give the total radiated power and possibly the directivity. Others allow one to measure, or infer, the amount of sound produced by various parts of the structure. Present concerns about machinery noise and noise reduction by design create special interest in techniques that allow one to scan the near field of the machine and determine noise radiated by various elements or surfaces.

The most commonly used technique is the reverberation method, in which the machine is placed in a room of fairly low absorption. In such a room, the reverberant mean square pressure $\langle p_R^2 \rangle$ is related to the radiated power Π_{rad}

$$\langle p_R^2 \rangle = \frac{4\Pi_{rad}\rho_o c}{R} \tag{3}$$

where $R = S\bar{\alpha}/(1-\bar{\alpha})$ is the "room constant", $\bar{\alpha}$ is the absorption coefficient in the room, and S is its interior area. The measurement of $\langle p_R^2 \rangle$ can only be done reliably when the wavelength of sound is less than half of a typical room dimension and if the source does not contain dominating pure tone components.

Another well known procedure employs a reflection free, or anechoic, room to measure the direct field $\langle p_D^2 \rangle$ from the radiator, which is related to the radiated power by

$$\langle p_D^2 \rangle = \frac{\Pi_{rad}\rho_o c}{4\pi r^2} Q \tag{4}$$

where Q is the directivity function and r is the distance from the "acoustic center" of the source. Since one does not know where the acoustic center of a source is, r must be large enough so that such uncertainties don't matter. Typically, r must be greater than the largest dimension of the source for the measurement to be in the "far field" or Frauenhofer zone of the radiator. This measurement technique allows one to determine the directivity function Q and the total power by an integration over solid angle.

A variation of the methods in the two preceding paragraphs is the "window" technique in which the machine is wrapped and then various positions of the machine are exposed. In this way, the contributions to the total noise power (and directivity) can be determined, if we assume that the process of wrapping does not disturb the relative roles of various elements in sound radiation. A sketch of a machine with its wrappings undergoing this process is shown in fig. 8. This procedure is conceptually simple, but the process of wrapping and unwrapping and the repetition of the sound measurements for each case can get quite time consuming and cumbersome.

The direct, or free field, method is essentially a measurement of sound intensity with a microphone. An intensity measurement close to the machine surface requires both a velocity and pressure measurement. The velocity measurement may be done using either a pressure gradient microphone or an accelerometer mounted to the surface of the structure, as shown in fig. 9. The required filtering, multiplication, and time averaging can be done by either analog or digital methods. The principal challenge is making sure that relative phase variations in the pressure and velocity channels are kept to a minimum.

Measurements of the intensity near the surface of a supported plate using a microphone-accelerometer scheme are shown in figs. 6 and 7. These measurements are in good agreement with the theoretical predictions shown in these figures, but these examples demonstrate one of the difficulties of applying this method. Since there are some areas of sound generation and others of sound absorption, a correct assessment of total, radiated-sound power requires

a very careful and accurate scan of the surface. Also, the requirement for two, phase-matched, measurement channels adds to the complication. Thus, its practical utility in measuring machine component noise is likely to be fairly limited.

Since measurements at the surface of a structure to determine relative sound generation of its various parts are so desirable, schemes have been developed which, although they lack a strict theoretical basis, are used because they seem to give useful answers, are very easy to implement, and the results are easy to interpret. The methods are the near field pressure scan and the acceleration, or radiation, efficiency method.

The near-field pressure scan takes note of the fact that the intensity of sound in a plane wave is $\langle p^2 \rangle / \rho_o c$ and in a diffuse field (over solid angle 2π) is $\langle p^2 \rangle / 2\rho_o c$ to assert that near a machine surface the intensity is $\langle p^2 \rangle / \delta \rho_o c$, where δ is to be determined. Each part of the machine has an area S_i and, consequently, the total power is

$$\Pi_{rad} = \sum_i \frac{\langle p_i^2 \rangle \; S_i}{\delta \rho_o c} \tag{5}$$

If Π_{rad} is known from a reverberant measurement, δ is determined. Most studies suggest a value of 4 for δ. In fig. 10, we show the total, sound-power output measured for a consumer sewing machine and the relative contribution to the total radiated power from its various surfaces as determined by this method. Also shown are the iso-pressure contours on this machine for the 500-Hz octave band.

The radiation efficiency method assumes that the sound radiation is dominated by vibrating structure. Mean square acceleration values $\langle a_i^2 \rangle$ are determined for various parts of the structure, and the radiated power is determined by a variant of eq. (2),

$$\Pi_{rad} = \sum_i \frac{\langle a_i^2 \rangle \; S_i}{\omega^2} \rho_o c \; \sigma_{rad,i} \tag{6}$$

One can assume that $\sigma_{rad,i}$ is the same for all surfaces and determine its value by a measurement of total radiated power. Then, the relative sound produced by each part of the machine is proportional to its contribution $\langle a_i^2 \rangle S_i$. This has been done for the sewing machine, and the result is shown in fig. 11. Obviously, this technique does not rank the sound output of the various elements in the same way that the pressure method does.

Of course, this last method can be improved by using the ideas presented in the section on "Simple Radiators" to make better estimates of

$\sigma_{rad,i}$. However, on balance, this vibration technique has several drawbacks compared to the pressure method. There is generally more variability to the acceleration field than in the pressure field so that an average is more difficult to determine. There is only a single, unknown parameter in the pressure method (δ) compared to several ($\sigma_{rad,i}$) in the acceleration method. Also, the acceleration method requires that structural vibration dominate the sound generation process, while no such assumption is made in the pressure method.

Clearly, more research and applications studies are required to define the basis for and limitations of these simplified methods for determining the sound produced by various parts of a machine or structure. Moreover, there is good reason to carry out these studies because of the importance of such measurements in developing noise reduction treatments for machines, particularly in the important area of redesign for reduced noise emission.

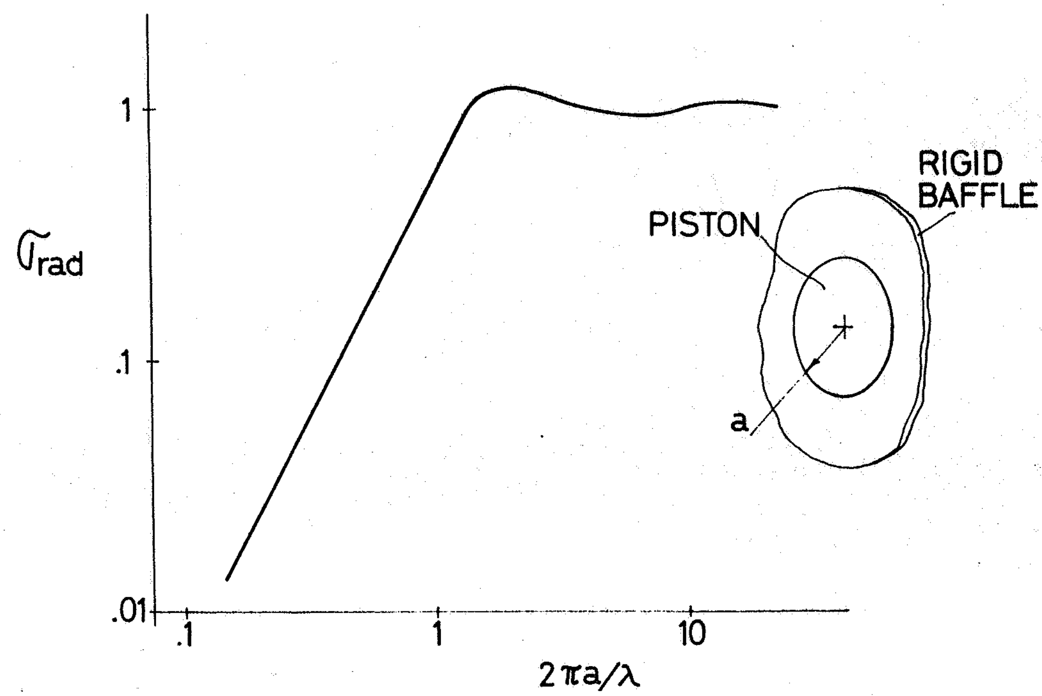

Figure 1.- Sketch of vibrating piston and theoretical radiation efficiency.

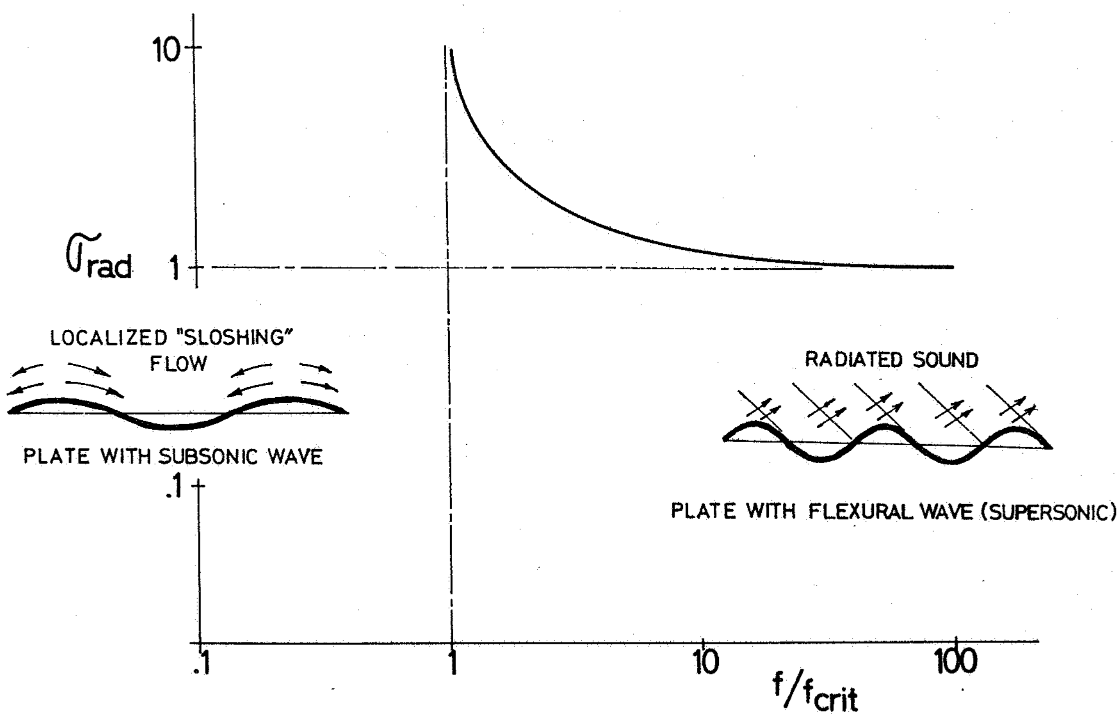

Figure 2.- Sketch of radiation of sound by vibrating plate, associated air motion, and resulting radiation efficiency.

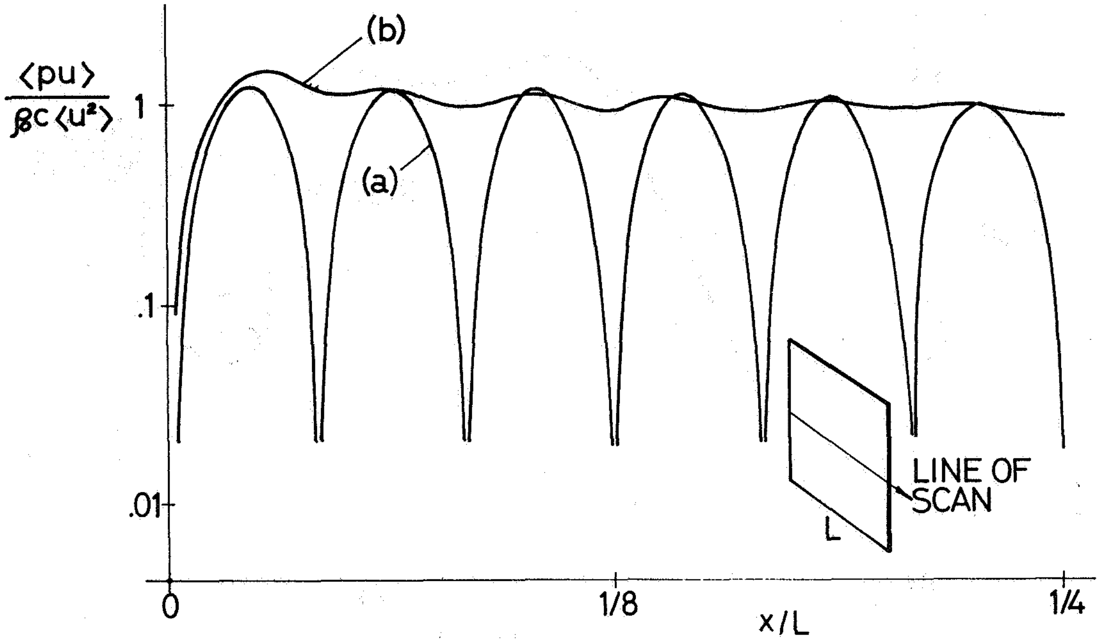

Figure 3.- Normal component of intensity vector at surface of vibrating, simply-supported plate above its critical frequency. (a) Single mode, (b) multi-modal.

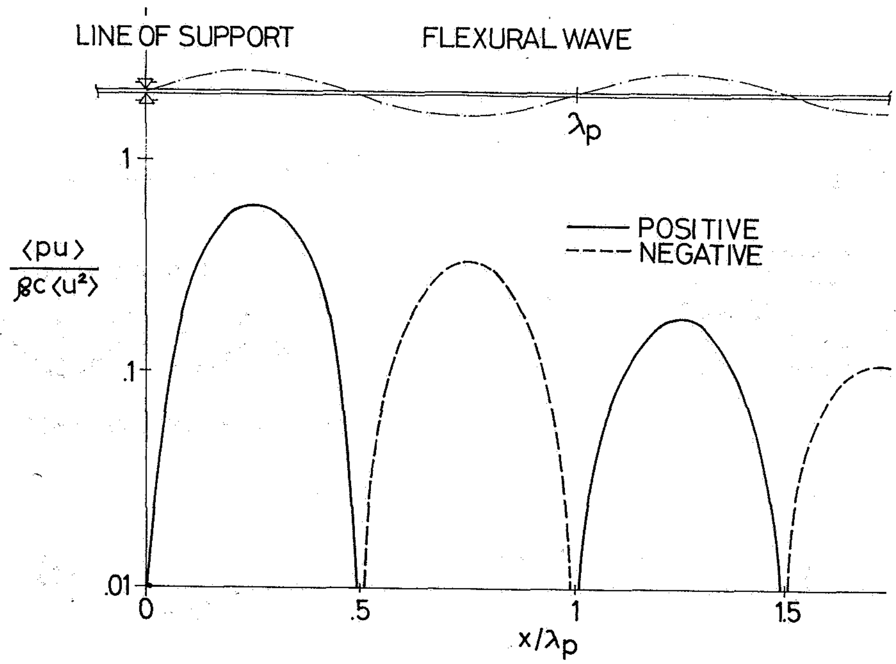

Figure 4.- Intensity near a line of support on an infinite plate showing regions of positive and negative intensity.

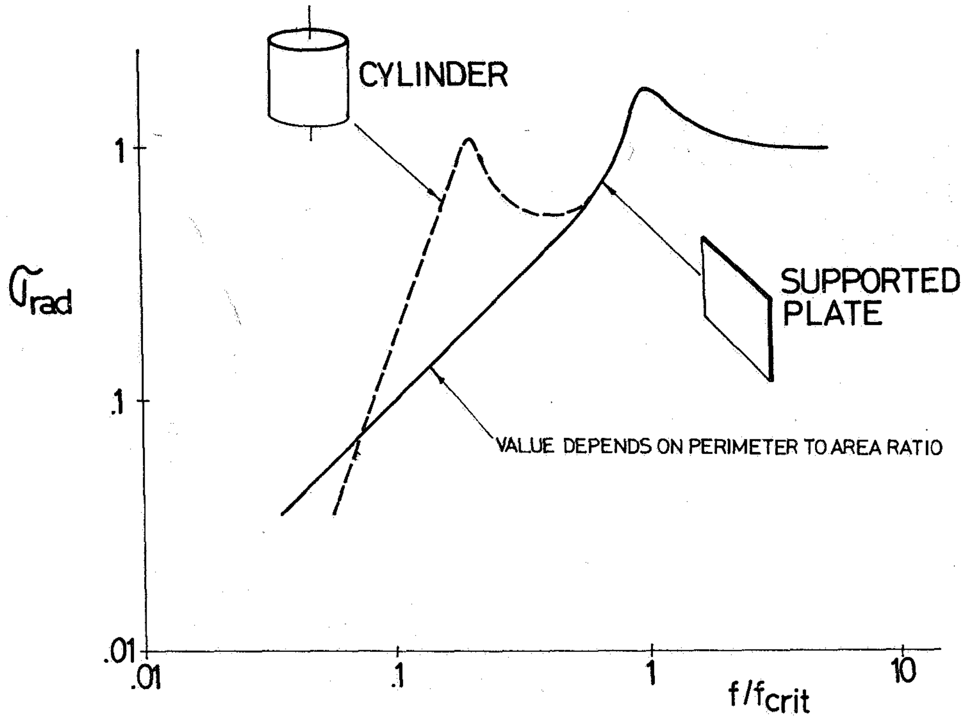

Figure 5.- Radiation efficiency of finite supported plate and cylinder of same area.

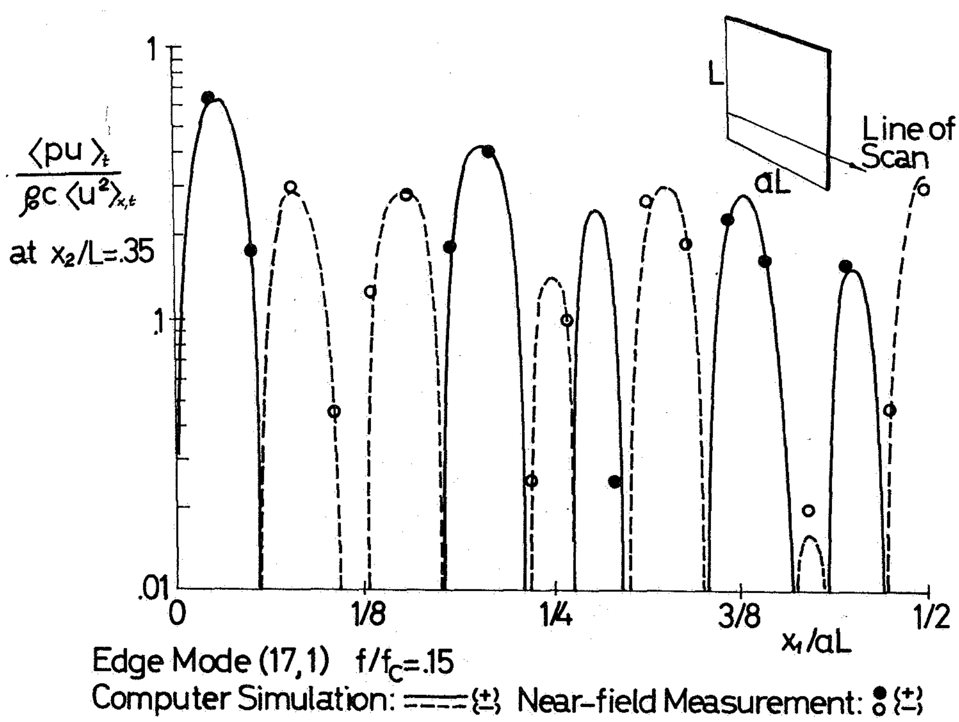

Figure 6.- Intensity scan across mid-section of rectangular supported plate showing region of sound absorption near the edge.

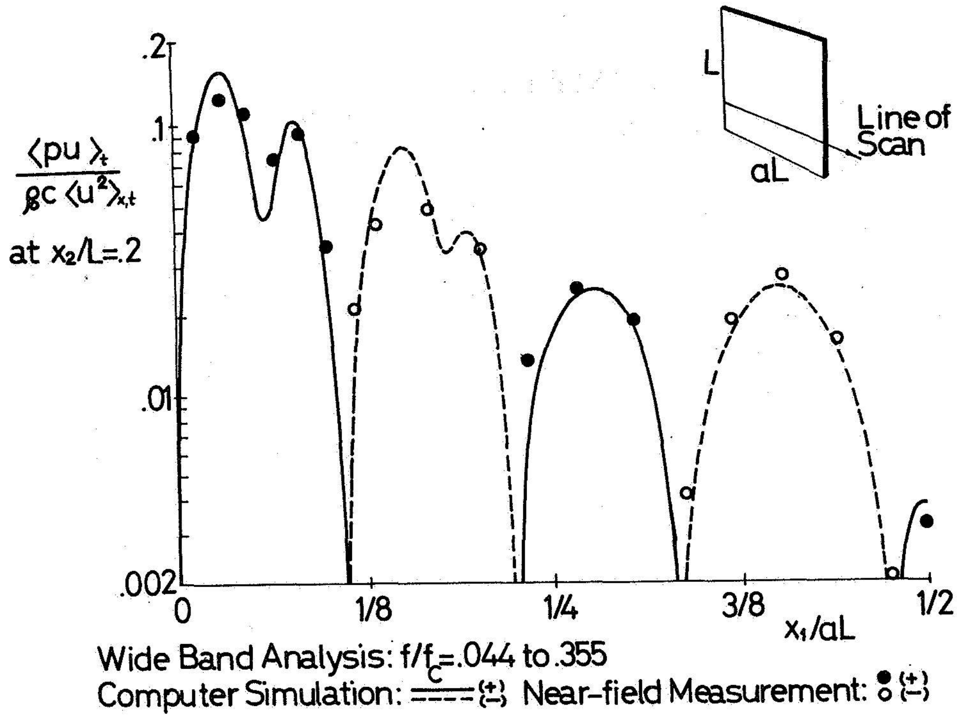

Figure 7.- Measured intensity along mid-section of simply-supported plate below the critical frequency.

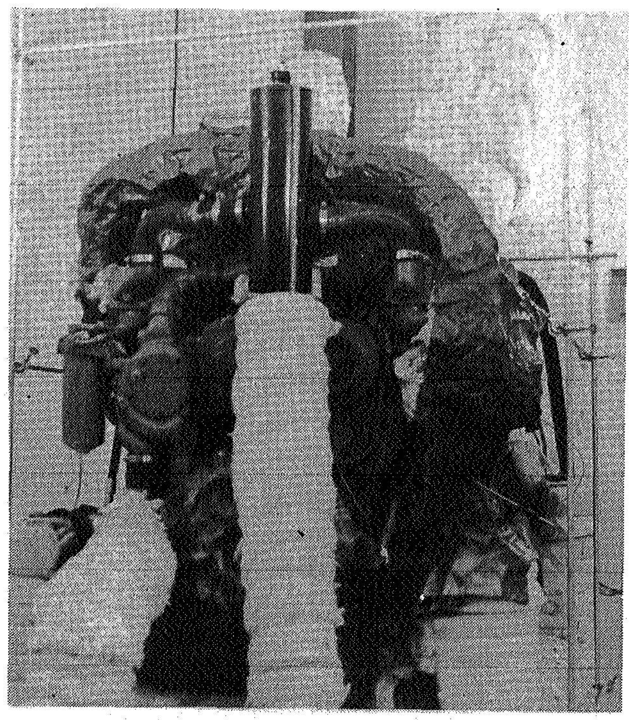

Figure 8.- In the window method, various parts of a wrapped machine are exposed for measurements of noise using either reverberant or anechoic methods.

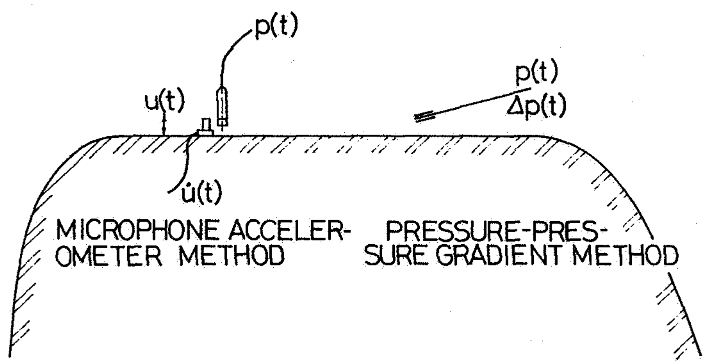

Figure 9.- Two methods for measuring the local sound intensity at the surface of a structure.

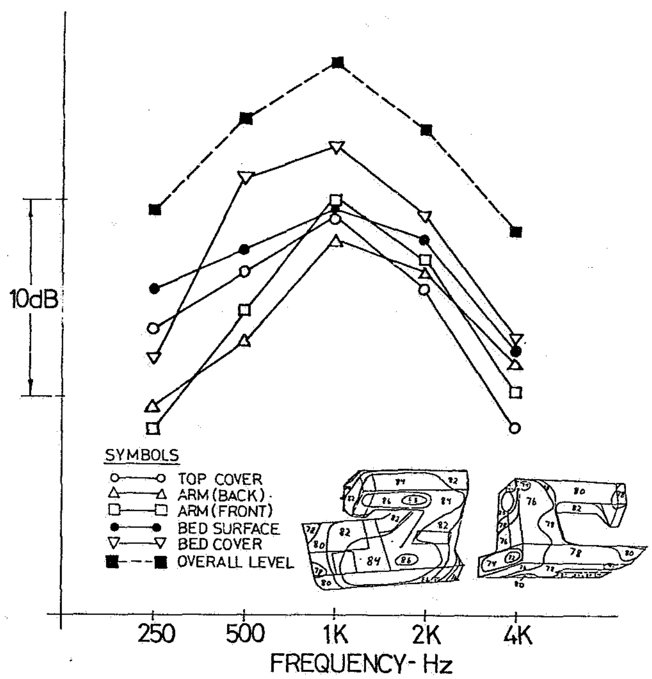

Figure 10.- Total and relative contributions of various machine surfaces to sound power radiated by a sewing machine as determined by the near field pressure method.

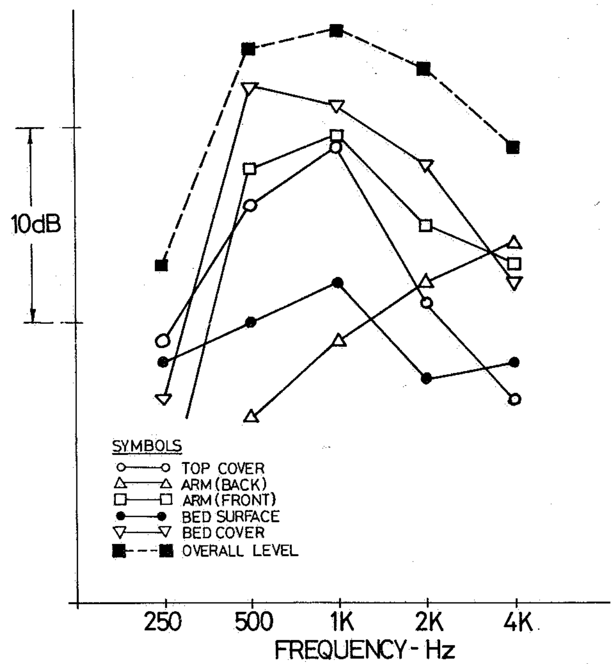

Figure 11.- Total and relative contributions to sound power radiated by sewing machine as determined by the acceleration method, assuming a uniform radiation efficiency for all surfaces.